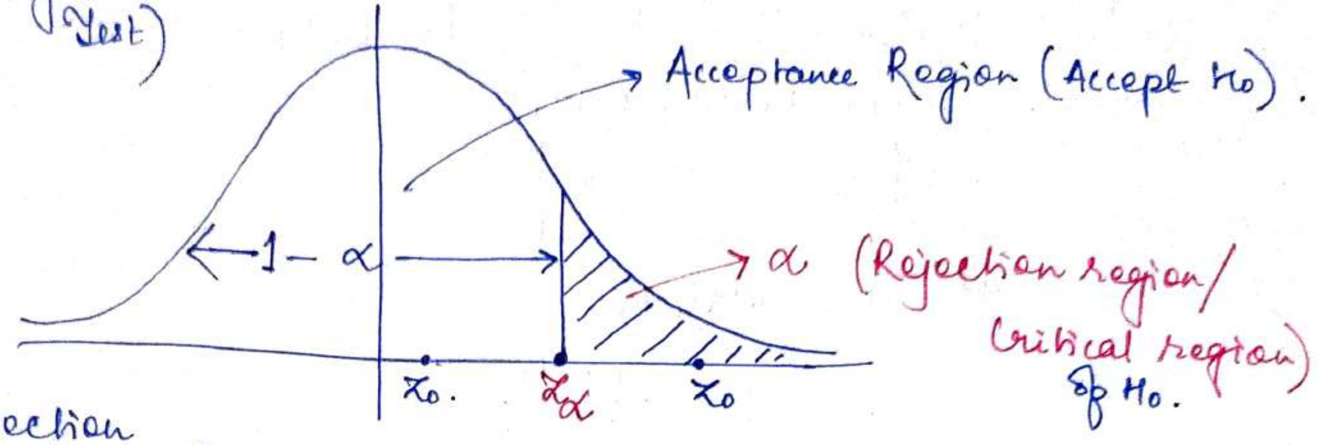
(13) One-Tailed Test.

(Right-tailed Test)

$H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$



Rejection ~~Acceptance~~ Region:- $P(z > z_\alpha) = \alpha$

Acceptance Region:- $P(z < z_\alpha) = 1-\alpha.$

Approach 1: (Without P-value)

(a) for 5% level of significance; i.e. $\alpha = 0.05$.

Acceptance Region $= P(z < z_\alpha) = 1 - \alpha.$

$= 1 - 0.05 = 0.95$

from z-table / normal table:

$\boxed{z_\alpha = 1.65}$ → for area value $= 0.95$.

from test-statistic:

$$z_{calc} = z_0 = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}.$$

* If $z_0 > 1.65$ → Reject H_0. ✓

$z_0 < 1.65$ → Accept H_0.

(b) for 1 % level of significance :-

i.e. $\alpha = 0.01$

$$P(z < z_\alpha) = 1 - \alpha = 1 - 0.01 = 0.99$$

from normal table:-

$$\boxed{z_\alpha = 2.33}$$

from test - statistic:-

$$z_0 = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

→ If $z_0 > \underbrace{2.33}_{z_\alpha}$ ⇒ Reject H_0.

→ If $z_0 < \underbrace{2.33}_{z_\alpha}$ ⇒ Accept H_0.

Approach-2: P-value approach.

for Right tailed test: $H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$.

Test statistic: $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = z_0$.

$$\text{P-value} = P(Z > z_0)$$

~~{If z_0 is (+)ve}~~

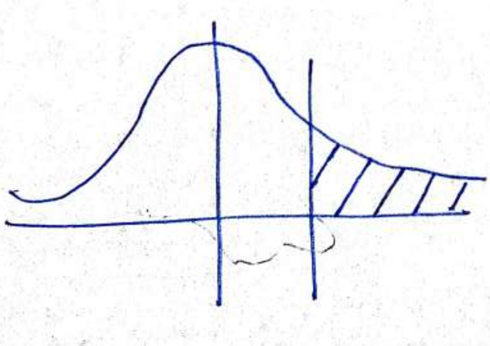
$$= 1 - P(Z \le z_0)$$

IMP

for Right tailed test

$$\left[\text{P-Value} = P(Z > z_0) = 1 - \underbrace{P(Z \le z_0)}_{\text{from normal table.}} \right]$$

= Area of the shaded region.



Problem:- A random sample of 100 recorded deaths (8)
Ex:- 10.9 in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years?

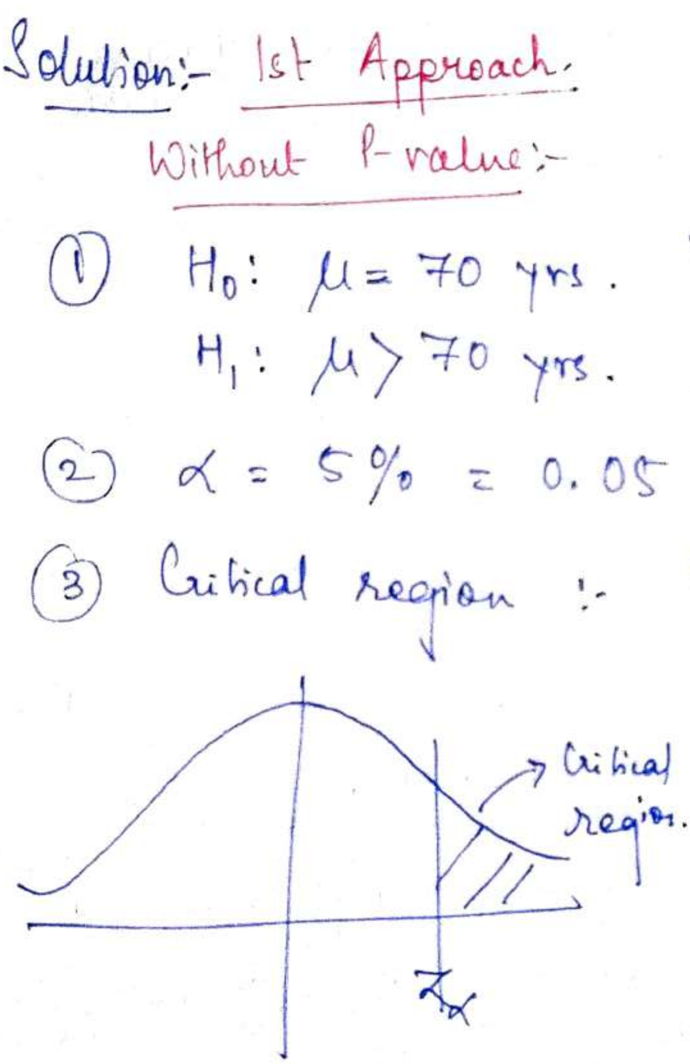
Solution:- 1st Approach.

Without P-value:-

(1) $H_0: \mu = 70$ yrs.
$H_1: \mu > 70$ yrs. } 1 tailed (right-tailed)

(2) $\alpha = 5\% = 0.05$

(3) Critical region :- $z > z_\alpha$



i.e. $z > \underline{1.65}$ (z_α)

where,

$$z = z_0 = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}.$$

(4) We have:- $\bar{x} = 71.8$ yrs., $\sigma = 8.9$ yrs., $n = 100$.

$$\therefore z_0 = \frac{71.8 - 70}{\frac{8.9}{\sqrt{100}}} = \frac{1.8}{0.89} = \underline{\underline{2.02}}.$$

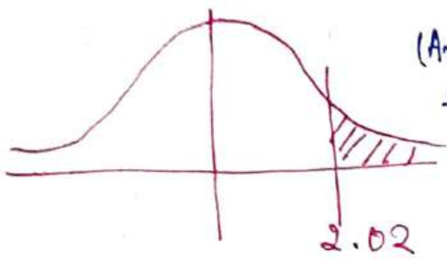
(5) Decision:- $\because z_0 = \underline{2.02} > z_{0.05} = \underline{1.65}$

$\therefore$ Reject H_0 & conclude that the mean life span today is greater than 70 years.

Approach-2. (P-value)

Since, it is a right-tailed test.

$\therefore$ P-value $= P(Z > Z_0)$



(Area of the shaded region).

$= P(Z > 2.02)$

$= 1 - P(Z \leq 2.02)$ → Normal table.

$= 1 - 0.9783.$

$= 0.0217.$

$\because$ P-value $= 0.0217 < \alpha = 0.05$

$\therefore H_0$ is rejected.

$\therefore$ We conclude that the mean life span today is greater than 70 yrs. (i.e. Accept H_1).

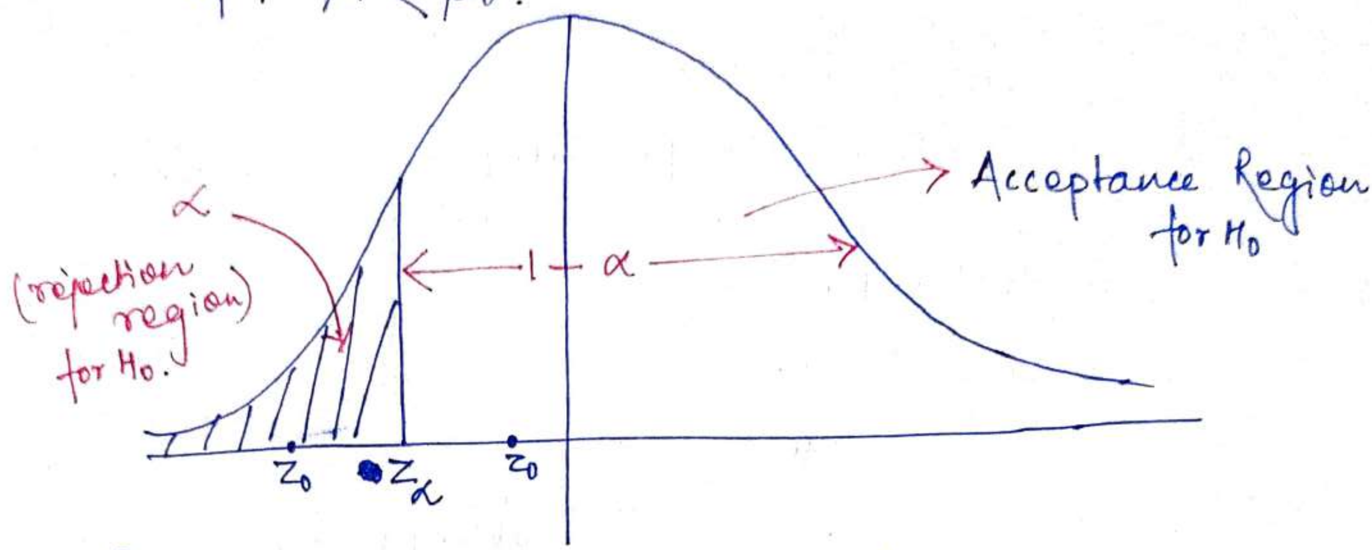
Problems for Practice:- (Exercises Problem: 10.22)

In the American Heart Association journal "Hypertension", researchers report that individuals who practice Transcendental Meditation (TM) lower their blood pressure significantly. If a random sample of 225 male TM practitioners medidate for 8.5 hours per week with a standard deviation of 2.25 hours, does that suggest that, on average, men who use TM meditate more than 8 hours per week? Take level of significance as 1 % and quote a P-value in your conclusion.

(c) One-Tailed Test :- (Left-tailed Test)

Eg:- $H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$.



Rejection region : $P(Z < z_\alpha) = \alpha$

Approach 1 : (without P-value)

(a) for 5% level of significance : i.e. $\alpha = 0.05$

Rejection region $= P(z < z_\alpha) = \alpha = 0.05$

from z-table / normal table :-

For $\alpha = 0.05$, corresponding $z_\alpha = -1.65$

$\Rightarrow \boxed{z_\alpha = -1.65}$

Now, $z_{calc.} = z_0 = z_{stat} = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

z_α

* If $z_0 > -1.65 \rightarrow$ Accept ~~Reject~~ H_0. ✓

If $z_0 < -1.65 \rightarrow$ Reject H_0. ✓

(b) for 1 % level of significance :- (i.e. $\alpha = 0.01$)

$$P(z < z_\alpha) = \alpha = \text{Rejection region.}$$
$$= 0.01.$$

from normal table :-

for $\alpha = 0.01$, corresponding $\boxed{z_\alpha = -2.33}$.

from test - statistic :-

$$z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

z_α

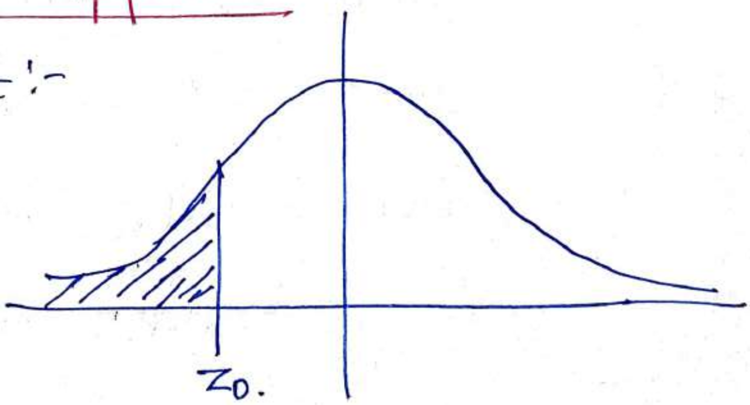
* If $z_0 > -2.33 \Rightarrow$ Accept H_0.
* If $z_0 < -2.33 \Rightarrow$ Reject H_0.

Approach - 2. p - value Approach :-

for Left - tailed test :-

$H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$.



Test - statistic :-

$$z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

P - value = Area of the shaded region.

$$= P(z < z_0)$$

Problem:- Exercises (Q. 10.19)

In a research report, Richard H. Weindruch of the UCLA Medical School claims that mice with an average life span of 32 months will live to be about 40 months old when 40% of the calories in their diet are replaced by vitamins and protein. Is there any reason to believe that $\mu < 40$ if $\boxed{\text{64 mice}}$ that are placed on this diet have an average of 38 months with a standard deviation of 5.8 months? Use a P-value in your answer. Take $\alpha = 5\%$ level of significance.

Answer:-

1st Approach:- (without P-value)

① $H_0 : \mu = 40$
$H_1 : \mu < 40$.

~~Given: population mean~~

Note:- V.V. Imp.

Population mean i.e. $\mu_0 = 40$.

But population standard deviation is unknown.

By rule, when population S.D is unknown that time we use t-test.

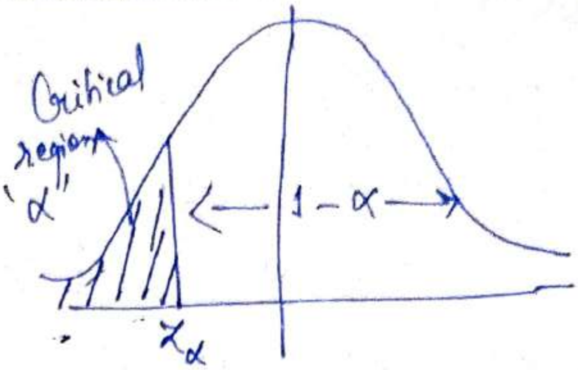
But here we can't use t-test, as t-test is used for sample size ≤ 30.

And we are having in this question, sample size $n = 64$.

So, we use Z-test instead of t-test & sample standard deviation will act as population standard deviation.

2. $\alpha = 5\% = 0.05$.

3. Critical Region/ Region of rejection of H_0 is $z < z_\alpha$



i.e. $\boxed{z < \underbrace{-1.65}_{z_\alpha}}$

4. We have:-

- population mean $= \mu_0 = 40$.
- sample mean $= \bar{x} = 38$
- population S.D $= \sigma = 5.8$ (not s as it is a special case as mentioned in the previous page)
- $n = 64$.

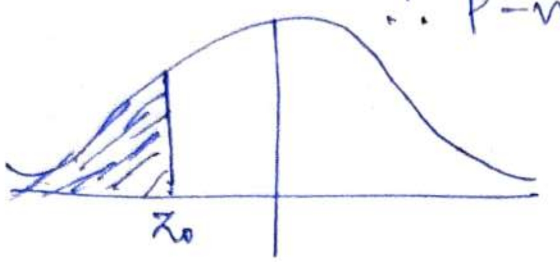
5. $$z_0 = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{38 - 40}{\frac{5.8}{\sqrt{64}}} = \frac{-2}{\frac{5.8}{8}} = \frac{-16}{5.8} = -2.76.$$

6. Decision:- $\because z_0 = -2.76 < z_\alpha = -1.65$

$\therefore$ We reject H_0 and conclude that yes $\mu < 40$ if 64 mice are placed on a diet in which 40% of the calories are replaced by vitamins & proteins.

Using P-value Approach.

Since, it is a left-tailed test



$\therefore$ P-value = Area of the shaded region

$= P(z < z_0)$

$= P(z < -2.76)$

$= 0.0029$ (from Normal table).

Decision: $\because$ P-value $< \alpha = 0.05$

$\therefore H_0$ is rejected and H_1 is accepted.

i.e. Yes, $\mu < 40$, if 64 mice are placed on a diet in which 40 % of the calories are replaced by vitamins & proteins.

Problem for Practice: Exercise Problem:- 10.20

A random sample of 64 bags of white chedar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.