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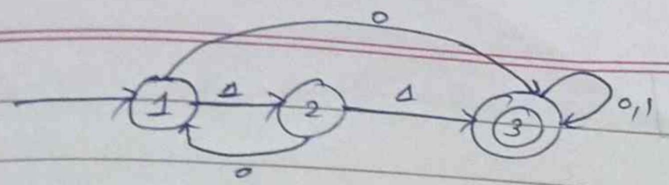
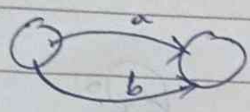
Q)



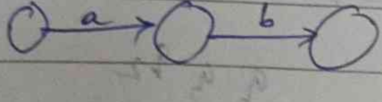
$R_{13} = ?$

(1)



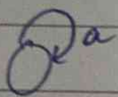
$RE = a + b$

(2)

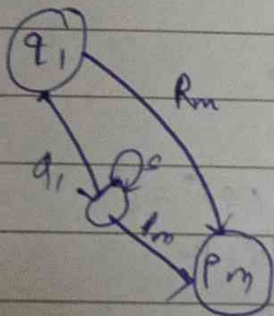


$RE = a \cdot b$

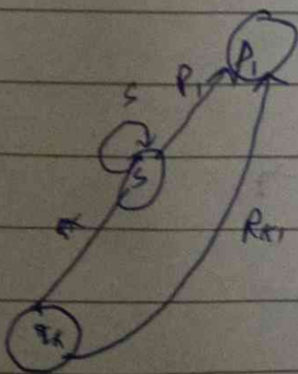
(3)



$RE = a^*$



$R_{1m} + Q_1 S^* P_m$

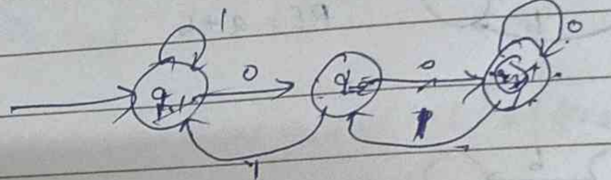


$R_{k1} + Q_k S^* P_1$.

$\boxed{R+QS^*P}$

DP IDP

d) Construct RE from DFA using state Elimination method -



$q_2\ q_3\ q_2$

Eliminate q_2.

(i) Case 1: Path in $q_1 \to q_2 \to q_3$.

$R = \phi,\ Q = 0,\ S = \phi,\ P = 0$

$= \phi + 0\cdot(\phi)^*\cdot 0$ $\left[\because (\phi)^* = \varepsilon,\ \varepsilon R = R\right.$

$= \phi + 0\cdot 0$

$= 00$

$q_1 \xrightarrow{00} q_3$

Case : Path in $q_3 \to q_2 \to q_1$

$R = \phi,\ Q = 1,\ S = \phi,\ P = 1$

$= \phi + 1\cdot(\phi)^* P$

$= \phi + 1\cdot 1$

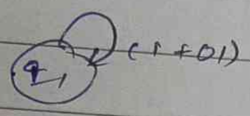
$= 11$

(iii) Case 3 : Path in $q_1 \rightarrow q_2 \rightarrow q_1$

$R = 1,\ Q = 0,\ S = \phi,\ P = 1$

$= 1 + 0 \cdot (\phi)^{2} \cdot 1$

$= 1 + 01$

$=$ 

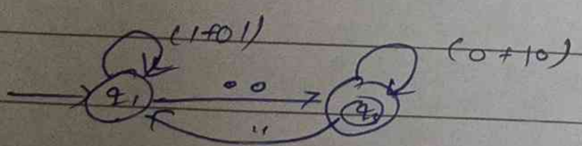
(iv) Case 4 : $q_3 \longrightarrow q_2 \longrightarrow q_3$

$R = 0\ ;\ Q = 1,\ S = \phi,\ P = 0$

$= 0 + 1(\phi)^{*} \cdot 0$

$= (0 + 10)$

Combine all 4 cases

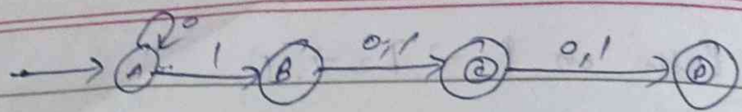


$R = (1+01),\ S = 00,\ U = (0+10),\ T = 11$

$RE = (R + SU^{*}T)^{*}SU^{*}$

$= ((1+01) + 00(0+10)^{*}11) \cdot 00(0+10)$

8)



4.02.2025

classmate
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* Pumping lemma for Regular Languages :-

* Pumping lemma is used to prove that a language is not regular.
* It cannot be used to prove that a language is regular.

Theorem : If A is a ~~ter~~ Regular language, then A has a pumping length 'p' such that any string 's' where $|s| \geq p$ may be divided into three parts $s = xyz$ such that the following conditions must be true.

1) $xy^iz \in A$ for every $i \geq 0$

2) $|y| > 0$

3) $|xy| \leq p$.

In simple words, if a string y is 'pumped' or insert any number of times, the resultant still remains in A.

Q) Prove the $L = \{a^p / p$ is a prime$\}$ is not regular.

Ans - $L = \{aa, aaa, aaaaa, a^7, a^{11} \ldots\}$

Step 1 :- Assume L is RL.

n is no. of states in the FA.

Step 2 :- $w = aaa$ $n=3$

x y z

(i) $|y| > 0 = |a| > 0$
$= 1 > 0$. True.

(ii) $|xy| \leq n \Rightarrow |aa| \leq 3$
$2 \leq 3$ T

$|w| \geq n$
$3 \geq 3$ True.

Step 3 :- $xy^iz \in L$

if $i=0$, then $xy^iz = a(a)^0a = aa \in L$.

if $i=1$, then $xy^iz = a(a)a = aaa \in L$.

if $i=2$, then $xy^iz = a(a^2)a = aaaa \notin L$

This contradicts our assumption. Hence L is not regular.

Q) PT. $L = \{0^n 1^n \mid n \geq 1\}$ is not regular.

~~Step1:~~ $L = \{01, 0011, 000111, \ldots\}$

Step1 :- Assume L is RL.

n is no. of states in the FA.

~~L is not RL.~~

Step2 :- $w = 0011$

y has only zero — (i) $w = 00\,11$ → x, y, z

y has only 1 — (ii) $\boxed{00}11$ → x, y, z

y has both 0 and 1 — (iii) $w = 0\boxed{01}1$ → x, y, z

~~(i) w = 0011~~

(i) y has only 0's, i.e. $y = 0^k$

$w = 0^n 1^n$

$x = 0^{n-k}$, $y = 0^k$, $z = 1^n$

Step3 :- $xy^iz \notin L$

if $i = 0$ then $xy^iz = 0^{n-k}(0^k)^0 1^n$

$= 0^{n-k} 1^n \notin L$

$= n-k \neq n$.

(ii) Y has only 1's, i.e. $y = 1^j$.

$$W = 0^n 1^n$$

$x = 0^n \quad y = 1^j \quad z = 1^{n-j}$

<u>Steps</u>:- $xy^iz \notin L$

~~If y~~

If $i=0$ then $xy^iz = 0^n \cdot (1^j)^0 \cdot 1^{n-j}$

$= 0^n \cdot 1^{n-j} \notin L$

$= n \neq n-j$.

(iii) Y has both 0's and 1's, ~~y=~~ i.e. $y = 0^k 1^j$

$$W = 0^n 1^n$$

$x = 0^{n-k} \quad y = 0^k 1^j \quad z = 1^{n-j}$

<u>Steps</u>:- $xy^iz \notin L$

if $i=2$ then $xy^iz = 0^{n-k} \cdot (0^k 1^j)^2 \cdot 1^{n-j}$

$= 0^{n-k} \cdot 0^k 1^j \cdot 0^k 1^j \cdot 1^{n-j} \notin L$.

5.02.2024

1 Q) P.T. $L = \{ a^n b^n c^n / n \geq 1 \}$ is not regular.

2 Q) P.T. $L = \{ ww^R / w \in \{a,b\}^* \}$ is not regular

eg:- $w = ab$
$w^R = ba$

3 Q P.T. $L = \{ ww / w \in \{a,b\}^* \}$ is not regular.

2) $L = \{ \varepsilon, ab, bb, abba, baab, abaaba \ldots \}$.

$W = \cancel{\varepsilon}\, ab$.

Step 1 :-

Assume L is a RL. Let n be the no. of states.

Step 2 :-

$W = \underbrace{a}_{x}\underbrace{b}_{y}\underbrace{ba}_{z}$ $\quad n = 4$

(i) $|W| \geq n = 4 \geq 4$ True

(ii) $|Y| > 0 = 1 > 0$ True.

(iii) $|x| \leq n = 2 \leq 4$ True.

Step 3 :-

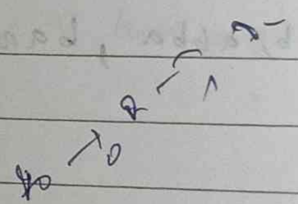
$xy^iz \notin L = a \cdot (b)^2 \cdot ba = a \cdot bb \cdot ba \notin L$

L is not regular.

3) P.T. $L = \{WW \mid W \in \{a,b\}^*\}$ is not regular.

Ans — $L = \{$

4) Design a NFA that doesn't end with 100 and 11.



* Properties of RL :-

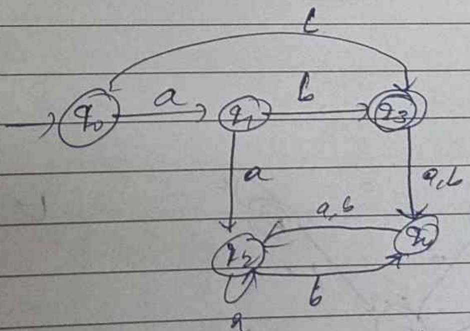
- Closure Properties - Useful tool for building complex Automata.
- Decision Properties - Deciding whether two automata define the same language.
- Proving languages not be regular - Pumping lemma is a powerful tool for proving certain languages are not regular.

1) Emptyness Problem :-

- Checking whether a given F.A. accepted empty language or not.

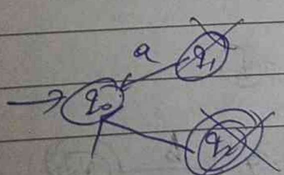
Algorithm :-

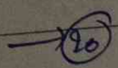
S1. Eliminate all inaccessible state from the given F.A.
S2. In the remaining FA if there is atleast one final state then one



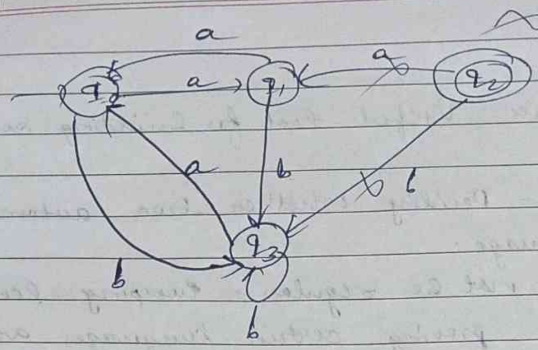
$L = \{b, ab\}$

Q2)





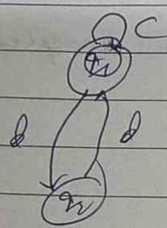
Q)



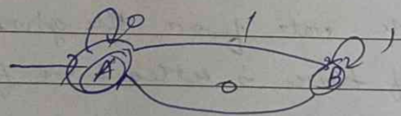
$L = \phi$

② Finiteness Problem :-

① Q)

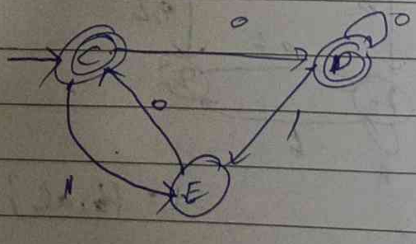


Q)



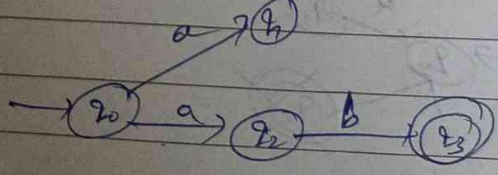
$L = \{0, 10, 1010, 10110, 101010, 10111010, \ldots\}$

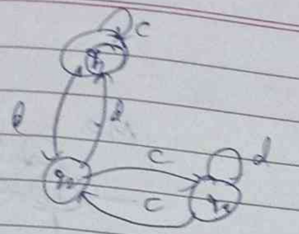
Q)

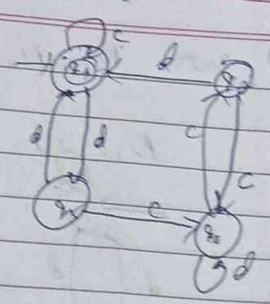


$L = \{0, 10, 10, 010, 10110, \ldots\}$

Q)



① 

② 

Equivalence of Two FA's :-

Comparision Method :-

Case 1 : M $\quad$ M'

$\uparrow$ $\quad$ $\uparrow$

q $\quad$ q'

FS $\quad$ FS

NF $\quad$ NF $\Big\}$ equivalent.

Case 2 : M $\quad$ M'

$\uparrow$ $\quad$ $\uparrow$

q $\quad$ q

F $\quad$ NF

NF $\quad$ F $\Big\}$ Not Equivalent.

$\Sigma = \{c, d\}$

Comparision table.

States	c	d
(q_1, q_4)	(q_1, q_4) $\uparrow \uparrow$ F F	(q_2, q_5) $\uparrow \uparrow$ NF NF
(q_2, q_5)	(q_3, q_6) $\uparrow \uparrow$ NF NF	(q_1, q_4) $\uparrow \uparrow$ F F
(q_3, q_6)	(q_2, q_4)	(q_3, q_6)
(q_2, q_4)	(q_3, q_6)	(q_1, q_4)

c = 0

b = 1

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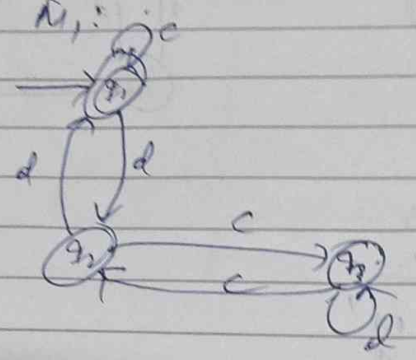
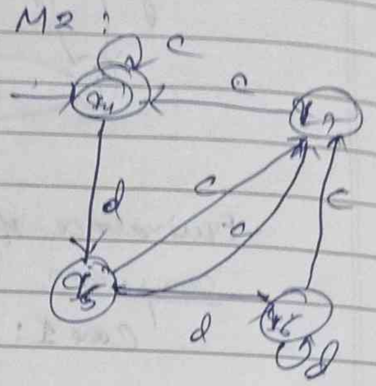
Q) State M_1, M_2.





Equivalence of Two FA's:-

Comparision Method:-

Case 1: M1 M2

q_1 q_4

FS FS

NF NF } Equivalent.