VIT-AP UNIVERSITY

QUESTION PAPER

Name of the Examination: FAT (Winter 2022 - 2023)

Course Code: MAT1002

Set Number: 7

Slot: 7

Duration: 120 min. (2 hours)

Course Title: Applications of Differentia[l and] Difference Equations

Total Marks: 60

Date of Exam: 16/06/2023 (FN)(

Instructions: Answer all the questions;

1. (a) Use Cayley-Hamilton theorem to find A^{-1} if [4 mark

$$A = \begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}$$

(b) A 12-volt battery (E) is connected to an LR-series circuit in which the inductanc (L) is 12 henry and the resistance (R) is 10 ohms. If the initial current is zer determine the current i using the following differential equation [4 mark

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}.$$

(c) Determine the inverse Laplace transform of [4 mark

$$F(s) = \frac{1}{s} \sin\left(\frac{1}{s}\right).$$

2. Using Frobenius method, solve the Bessel's differential equation [12 mark

$$xy'' + 2y' + xy = 0$$

expanded about the regular singular point $x_0 = 0$.

3. A cup of coffee has an initial temperature of 165°F, but cools to 155°F in one minute whe placed in a room with a temperature of 70°F. Let T_n be the temperature of the coffe after n minutes. The difference equation which describes the change in temperature the coffee from minute to minute is given by

$$T_{n+1} - T_n = k(T_n - S), \quad \text{where } S \text{ is the room temperature.}$$

Find the temperature of the coffee after 25 minutes. [12 mark

4. (a) Find the Z-transform of the following discrete-time signal **[6 marks]**

$$x(n) = n\left(\frac{1}{5}\right)^n u(n), \text{ where } u(n) \text{ is a unit step function.}$$

(b) Find the discrete-time signal whose Z-transform is given as **[6 marks]**

$$X(z) = \frac{1}{z(z-2)^2}$$

5. Using Z-transform method, find the response of a discrete time electrical signal given by the following difference equation **[12 marks]**

$$y(n+2) + y(n) = u(n),$$

where $u(n)$ is the unit step function. Assume that all the initial conditions are zero.

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QUESTION PAPER

Name of the Examination: FAT WIN 2022-23

Course Title: Applications of Differential and Difference Equations
Course Code: MAT1002
Set Number: 1
Duration: 120 min

Slot:
Date of Exam: 16 /06/2023 (AN) (A2)
Total Marks: 60

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1. (i) The coefficient matrix of a linear system of equations obtained by modelling a demand and supply problem is given by

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}.$$

Use Cayley-Hamilton theorem to find the inverse of the matrix.

(ii) A retired officer has some bank accounts, that pay interest but at the same time the owner intends to withdraw money at a constant rate. The differential equation for the balance y in the account at time t is given by $t^2y' + ty = 1, \quad t > 0, \quad y(1) = 2$. Find the solution of the initial-value problem.

(iii) Let $f(t) = (t^2 - 3t + 2)sin(3t)$ be the velocity function of a particle in time domain. Find the Laplace transform of $f(t)$ in frequency domain. (4 + 4 + 4 Marks)

2. Find the first four terms in each portion of the series solution around $x_0 = 0$ for the following differential equation

$$(x^2 + 1)y'' - 4xy' + 6y = 0.$$

(12 Mark)

3. The discrete-time input (x_n) and output signals (y_n) of a linear time-invariant system is related by the given below equation

$$y_{n+2} - 5y_{n+1} + 6y_n = n^2 + 3n + 2.$$

Compute the output y_n, without using the Z-transform. (12 Marks)

4. (i) Find the Z-transform of a real discrete time sequence $x[n] = n\ cos(n\theta)$.
(ii) Find Z $\{u_{n+2}\}$ if Z $\{u_n\} = \frac{z}{z-1} + \frac{z}{z^2+1}$. (6 + 6 Marks)

5. The system function that models a linear time-invariant system's output for each possible input is given by

$$Y(z) = \frac{z^2}{(z-3)(z-4)}.$$

Use the convolution method to find the impulse response of the system. (12 Marks)
(Hint: Z-transform of impulse response is the system function.)

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QP MAPPING

Q. No.	Module Number	CO Mapped	PO Mapped	PEO Mapped	PSO Mapped	Marks
1	1, 2 & 3	3	1,2			12
2	4	6	1,2,5			12
3	5	6	1,2,5			12
4	5	5	1,2			12
5	5	5	1,2			12

Q.5 The heat conduction inside the metal is related with the following difference equation

$$y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n = u(n)$$

where $u(n)$ is discrete unit step function defined by $u(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

and $y(0) = y(1) = y(2) = 0$. Determine the solution of the given difference equation.

QP MAPPING

Q. No.	Module Number	CO Mapped	PO Mapped	PEO Mapped	PSO Mapped	Marks
Q1	1,2,3	1,2,3,5	1,2,3,5,6,12			12
Q2	4	2, 3, 6	1,2			12
Q3	5	5,6	1,2,5			10
Q4	5	5,6	1,2,5			14
Q5	5	5,6	1,2,5			12

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QUESTION PAPER
Name of the Examination: Win 2022-23 - FAT

Course Code: MAT1002 **Slot:**
Course Title: Applications of Differential and Difference Equations
Set Number: 5 **Date of Exam:** 17/06/2023
Duration: 2 Hr **Total Marks: 60** (AN) (B2)

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1. (i) Let A be 3×3 matrix having eigenvalues 1,2,−1. Find the trace of the matrix $B = A - A^{-1} + A^2$. (4M)
   (ii) Find the governing differential equation for $y = e^x(A\cos x + B\sin x)$. (4M)
   (iii) If $L\{f(t)\} = \frac{e^{-\frac{1}{s}}}{s}$, find $L\{e^{-t}f(3t)\}$. (4M)

2. In an electro magnetic wave propagation, the following differential equation is obtained
$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0.$$
   Find the power series solution about zero. (12M)

3. In discrete time signal processing system, the resulting difference equation is
$$4u_{n+2} - 4u_{n+1} + u_n = 2^n + 2^{-n}.$$
   Find the general solution using complementary function and particular integral. (12M)

4. The dynamics of a discrete state system is modelled by the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = u_n$, with $y_0 = 0$, $y_1 = 1$ and $u_n = 1$ for $n = 0, 1, 2, 3, \cdots$. Using Z-transforms, find the response of the system. (12M)

5. (i) Verify convolution property of Z-transforms for the signals $u_n = n$ and $v_n = 1$.
   (ii) Obtain the sequence whose Z-transform is $U(z) = log\left(\frac{z}{z+1}\right)$. (8M+4M)

QP MAPPING

Q. No.	Module Number	CO Mapped	PO Mapped	PEO Mapped	PSO Mapped	Marks
1	1,2,3	1,2,3	1,2,3,5,6,12			12
2	4	4	1,2			12
3	5	5,6	1,2,5			12
4	5	5,6	1,2,5			12
5	5	5	1,2			12

Q.2) A long, cylindrical wire has a non-uniform temperature distribution along its length. The temperature of the wire at any point x is given by the function $y(x)$, where x represents the distance from one end of the wire. The temperature distribution is modeled by the following differential equation: **(12 Marks)**

$$9x(1-x)y'' - 12y' + 4y = 0.$$

Determine the power series solution for $y(x)$ around the point $x = 0$.

Q.3) *a)* Find the Z-transform of the sequence $a_n = \frac{1}{n!}$. **(06 Marks)**

b) Use convolution theorem to find the inverse Z-transform of the following function

$$X(z) = \frac{z^2}{(z-a)(z-b)}.$$

(06 Marks)

Q.4) The price of a commodity u_n for time n satisfies the following difference equation

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n.$$

Solve the above difference equation by using Z-transformation(u_n denotes today's price and u_{n-1} represents yesterday's price of the commodity). **(12 Marks)**

Q.5) A light elastic string streched between two fixed nails 160 cm apart, carries 15 loads of weight 50 gm each at equal intervals and the resulting tension is 200 gm weight. Show that the sag at the mid points ($i.e. y_8$) is 80 cm. The difference equation is given as

$$y_{i+1} - 2y_i + y_{i-1} = -\frac{hP_i}{T}.$$

Where h is the length of the intervals, P_i is the load at the i^{th} point and T is the tension at i^{th} point. **(12 Marks)**

Slot:

Set number: 6

Date of Exam: 19/06/2023 (F

(C2)

Duration: 120 min

Total Marks: 60

Instructions:

1. Assume data wherever necessary.
2. Any assumptions made should be clearly stated.

Q1. a) In furniture manufacturing, aluminum, brass, and copper are used to manufacture three different types of furniture A, B, and C. The requirement of metals (in Kg) for each type of furniture is given below.

	Furniture type A	Furniture type B	Furniture type C
Aluminium	1	1	1
Brass	1	2	3
Copper	1	2	5

Determine the number of furniture of each type that can be manufactured using 6, 10 and 12 Kg of Aluminium, Brass, and copper, respectively. **(6M)**

b) Biotransformation of an organic compound having concentration $x(t)$ can be modeled using ordinary differential equation $\frac{dx}{dt} + kx^2 = 0$, where k is the reaction rate constant. If $x = a$ at $t = 0$, then find the concentration of the organic compound. **(3M)**

c) Find the Inverse Laplace Transforms of $log\left(1 + \frac{1}{s}\right)$. **(3M)**

Q2. Use the method of Frobenius to find the solutions to the differential equation

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

in some interval $0 < x < R$. **(12M)**

Q3. In an experiment, the pressure of gas in a container is measured each second, and the pressure (in standard units) in n second is denoted by P_n. The measurements satisfy the recurrence relation,

$$P_{n+2} - 5P_{n+1} + 6P_n = n^2 + 3n + 2.$$

Find the explicit formula for P_n . **(12M)**

Q4. Suppose a cup of tea, initially at a temperature of 180°F, is placed in a room that is held at a constant temperature of 80°F. Moreover, suppose that after one minute, the tea has cooled to 175°F.

a) Write a difference equation, in standard first-order linear form, which describes the change in temperature of the coffee from minute to the minute using the above data and then solve it using Z- transforms.

b) What will the temperature be after 20 minutes? **(12M)**

Q5. a) The input signals of a linear time-invariant (LTI) system are given by $f(n) = n$ and $g(n) = n^2$. Evaluate the convolution between the two signals and also find the Z- transform to resultant. **(8M)**

b) Find the inverse Z- transform of $\left[\frac{z^2}{z^2+1}\right]$ **(4M)**

QP MAPPING

Q. No.	Module Number	CO Mapped	PO Mapped	PEO Mapped	PSO Mapped	Marks
Q1	1,2, 3	1,2, 3,6	1,2,3,5,6,12			12
Q2	4	4	1,2,5			12
Q3	5	6	1,2,5			12
Q4	5	5	1,2			12
Q5	5	5	1,2			12